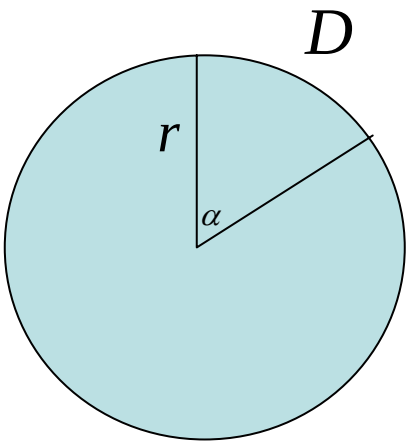
Radian measure

Angles are most often measured in degrees, arcminutes and arcseconds.

1 degree (1°) is 1/360 of a complete circle.
1 arcminute = 1/60 of a degree
1 arcsecond = 1/60 of a minute = 1/3600 of a degree

A circle has a circumference C =2πr so the distance around half a circle is πr and the distance around a quarter of a circle is 0.5πr etc ..

Let distance D be the distance around a circle spanned by α°
D is a fraction of the circumference C



$$\frac{D}{C} = \frac{\alpha}{360}$$

As $C = 2\pi r$ $$\frac{D}{2\pi r} = \frac{\alpha}{360}$$

So we can rearrange this formula to get

$$D = \frac{2\pi}{360} r\alpha$$

If we define a new unit of length the radian

where $\alpha^{c} = \frac{2\pi^{c}}{360^{\circ}} \times \alpha^{o}$

then the D formula becomes

$$D = r\alpha^{c}$$

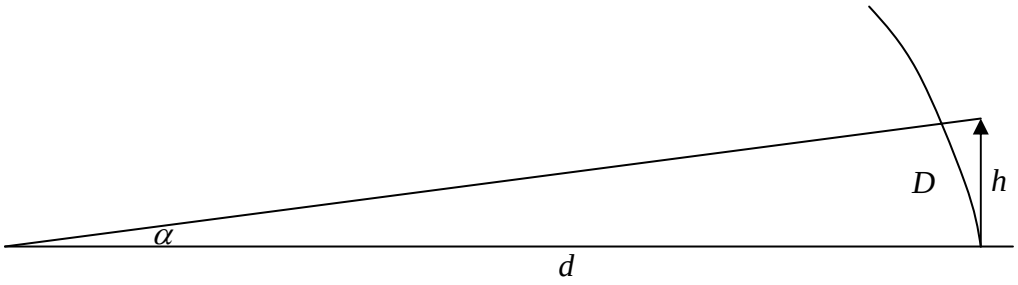
The superscript 'c' can be used to denote radians.

NOTE: To use this formula α MUST BE IN RADIANS.

Small angle approximation

The formula $D = r\alpha^{c}$ provides a way of estimating distances in certain circumstances.

Suppose a pole is stuck vertically into the ground a distance d away from an observer (see diagram).



If the angle α is small (less than 20 degrees) then the height h is very close to the distance D along the arc so that,

$$d = r$$

$$h \approx D = d\alpha^{c}$$

The smaller the angle is then the more accurate this approximation is.

If α is 1 degree (or 0.0174 radians) then D is within 0.01% of h.

Astrophysics Applications

In most astrophysical applications α is generally much less than 1 degree.

Instead of expressing α in radians, it is expressed in terms of arcseconds so a conversion factor is required.

As shown before, for α in degrees

$$D = \frac{2\pi}{360} d\alpha$$

2π radians is equivalent to 360 degrees

360 degrees is equivalent to $360 \times 60 \times 60$ arcseconds

For α in arcseconds

$$D = \frac{2\pi}{360 \times 60 \times 60} d\alpha$$

$$D = \frac{\alpha d}{206265}$$

D and d must have same units

(e.g. m, km, A.U., light years, parsecs)