ASTRONOMY 120: GALAXIES AND THE UNIVERSE
HOMEWORK # 9 SOLUTIONS
SPRING 2011

1. WIYN Telescope (Jedidah & Ana 51 points)

1.a. **The light from this protogalaxy contains the H-alpha emission line, whose rest wavelength (or emission wavelength)is 0.656 microns, in the red optical part of the spectrum. Calculate the wavelength at which we would observe the H-alpha line from this protogalaxy.**

$$\begin{aligned} \frac{\lambda_{obs}}{\lambda_{rest}} &= 1 + z \\ \lambda_{obs} &= \lambda_{rest} \times (1 + z) \\ &= 0.656\mu\text{m} \times (1 + 9.87) \\ &= 7.13\mu\text{m} \end{aligned} \tag{1}$$

1.b. **Can an optical-infrared telescope on earth's surface (like the WIYN telescope) observe its H-alpha line, if the telescope can observe from 2.2-0.3 microns?** If not, what kind of telescope would be required to observe it, and where should the telescope be located?

No, a telescope with a wavelength range between 2.2 and 0.3 μm will not be able to observe this red-shifted H-α line. In order to observe it you would need an infrared telescope capable of detecting longer wavelengths, which means htat you would need to use a space-based telescope (since the Earth's atmosphere is opaque to most infrared radiation). Spitzer Space Telescope would do nicely for this.

1.c. **The present separation of the Milky Way Galaxy and the Andromeda Galaxy (M31) is 2 million light years.** If you assume that the only motion of the galaxies is due to the expansion of the universe, how far apart were these 2 galaxies at the time that the light from the z=9.87 protogalaxy was emitted?

You know the present-day distance is 2 million light years. The conversion factor for distance is simply 1 + z:

$$\begin{aligned} d_{past} &= \frac{d_{now}}{1 + z} \\ &= \frac{2 \times 10^6\text{pc}}{1 + 9.87} \\ &= 184,000\text{ly} \end{aligned} \tag{2}$$

1.d. **How do these distances (at z=0 and z=9.87) compare to the present size of the Milky Way Galaxy?** What does this suggest about the size of the proto-Milky Way Galaxy at z=9.87?

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At z=0, the distance between the two galaxies is much greater than the size of the Milky Way today ($D_{MWA} \approx$ 100,000 light years) (3). At z=9.87, the distance between the two galaxies would be roughly the same as the size of the Milky Way (4).

$$\begin{aligned} \frac{d_{now}}{D_{MWA}} &= \frac{2 \times 10^6 \text{ly}}{1 \times 10^5 \text{ly}} \\ &= 20 \text{ times larger} \qquad (3) \\ \frac{d_{then}}{D_{MWA}} &= \frac{1.84 \times 10^5 \text{ly}}{1 \times 10^5 \text{ly}} \\ &\approx 2 \text{ timeslarger} \qquad (4) \end{aligned}$$

This suggests that the proto Milky Way must have been much smaller, since the two galaxies would probably have merged had they both been that large and so close together.

1.e. **How dense is the present Milky Way Galaxy, compared to the present average mass density of the universe? Assume that the Milky Way has a mass of $M = 10^{12}$ M$_\odot$ and the same radius you used in part d. Give your answer in the form of a ratio.** To calculate Milky Way density we assume it is spherical because the dark matter halo, which contains a large fraction of its mass, around it is roughly spherical.

$$\begin{aligned} \rho_{\text{MW now}} &= \frac{M}{V} = \frac{M}{\frac{4\pi}{3} r^3} \\ &= \frac{10^{12} M_\odot \times 2 \times 10^{30} \frac{\text{kg}}{M_\odot}}{\frac{4\pi}{3}\left(5 \times 10^4 \text{ ly} \times \frac{9.46 \times 10^{15} \text{ m}}{\text{ly}}\right)^3} \\ &= 4.5 \times 10^{-21} \text{ kg/m}^3 \qquad (5) \\ & \qquad (6) \end{aligned}$$

Looking up the average density of the universe today, we can set up a ratio

$$\begin{aligned} \frac{\rho_{\text{MW now}}}{\rho_{\text{universe now}}} &= \frac{4.5 \times 10^{-21} \text{ kg/m}^3}{2.4 \times 10^{-27} \text{ kg/m}^3} \\ &= 1.9 \times 10^6 \qquad (7) \end{aligned}$$

Present day Milky Way is 1.9×10^6 times more dense than the universe today.

1.f. **How dense is the present Milky Way Galaxy, compared to the average mass density of the universe at z=9.87? Give your answer in the form of a ratio.** Since size of the universe changes by a factor of $1+z$, volumes and densities change by a factor of $(1+z)^3$:

$$\begin{aligned} \rho_{past} &= (\rho_{now})(1+z)^3 \\ &= (2.4 \times 10^{-27} \text{ kg/m}^3)(1+9.87)^3 \\ &= (2.4 \times 10^{-27} \text{ kg/m}^3)(1284) \\ &= 3.1 \times 10^{-24} \text{ kg/m}^3 \end{aligned}$$

Setting up the density ratio:

$$\frac{\rho_{\text{MW now}}}{\rho_{\text{universe then}}} = \frac{4.5 \times 10^{-21}\ \text{kg/m}^3}{3.1 \times 10^{-24}\ \text{kg/m}^3} = 1.5 \times 10^3$$

Present day Milky Way is 1.5×10^3 times more dense than the universe at $z = 9.87$.

1.g. **What does the change in the density ratios in parts e and f tell you about the formation of structure in the universe?** Universe is becoming less dense because it expands, so at earlier times its density was more similar to the present day density of Milky Way, making the ratio $\rho_{\text{MW now}}/\rho_{\text{universe}}$ smaller. Structure like galaxies started forming from very small density enhancements with respect to surrounding area in the early universe (even before $z = 9.87$). Since then, the overdensities have grown in mass and density. The universe has grown in size, increasing the space between these overdensities and lowering its average density, so the ratio $\rho_{\text{MW}}/\rho_{\text{universe}}$ keeps getting larger.

1.h. **What was the temperature of the cosmic background radiation at the time corresponding to z=9.87?** Cosmic background radiation measured today peaks at wavelength of $\lambda = 1.06 \times 10^{-3}$ m, which corresponds to radiation of blackbody with temperature of $T = 2.73$ K. At redshift $z = 9.87$, wavelengths of CMB photons were shorter by a factor of $(1+z)$, corresponding to blackbody temperature higher by the same factor:

$$T_{\text{then}} = T_{\text{now}} \cdot (1+z) = 2.73\ \text{K} \cdot 10.87 = 29.7\ \text{K}$$

The temperature of cosmic background ratiation at the time corresponding to z=9.87 was 29.7 K.

2. Red-Shifted Cosmic Background (Arthur - 21 points)

2.a. **What was the peak wavelength in the spectrum of the Cosmic Background radiation at a time in the universe corresponding to a redshift z=1100?** The present temperature of the CMB is roughly 2.7K. At a redshift of 1100, the temperature should be $\approx (1+z) * T_o$ or $\approx$ 2973K. Then, to find the maximum wavelength, we use Wien's Law:

$$\lambda_{max} = \frac{0.29\text{cm K}}{T} = \frac{0.29}{2973} = 9.66 \times 10^{-5}\text{cm} = 966\text{nm} = 9660\mathring{A}$$

2.b. **What is the wavelength of a photon which has just enough energy to ionize a Hydrogen atom? [There is no calculation for part b. Just look up answer. See Section 5-8].** The text gives the value as 91.1 nm.

2.c. **Given that these are different, why is the Cosmic Background radiation at z=1100 able to ionize all the Hydrogen? [HINT! The ratio of the number of cosmic photons to protons is relevant!] [ANOTHER HINT!! Do all the photons have the same energy?] [NO MORE HINTS!!!]** First, one should note that the peak wavelength of the CMB does not have a short enough wavelength (and therefore a high enough energy) to ionize hydrogen in its ground state. Thus, on the surface, it appears the the CMB at z=1100 can NOT ionize hydrogen. However, the CMB exhibits a blackbody spectrum, thus, it has photons at all wavelengths. Most photons are at λ_{max}, but there are some at other wavelengths, including those short enough to ionize hydrogen (those in the UV part of the spectrum). Because the ratio of photons to protons is so high, there are enough UV photons in the tail end of the blackbody CMB spectrum to ionize all the hydrogen at z=1100.

3. Doppler Shifted Cosmic Background (John - 28 points)

3.a. **What is the peak wavelength of the cosmic background radiation for an observer at rest with respect to the cosmic background radiation? Calculate this (to 4 signficant figures) from the present temperature of the cosmic background radiation and Wien's Law.** Since the changes will be small in this problem, we will want 4 significant figures in our answer to see the differences. To do this, we will need to *start* with 4 significant figures in the numbers we use. Wien's law will give us the peak wavelength:

$$\begin{aligned}\lambda_{peak} &= \frac{b}{T} \\ &= \frac{2.898 \times 10^{-3}[\text{mK}]}{2.725[\text{K}]} \\ &= 1.063 \times 10^{-3}[\text{m}]\end{aligned}$$

3.b. **Suppose you live in a galaxy which is orbiting at high speeds through a cluster, and is therefore moving at speeds of 1555 km/s with respect to the cosmic background radiation. [Ignore the effects of the rotation of the galaxy.] What is the peak wavelength of the cosmic background radiation measured from a point on the sky which is in the direction of motion?** When we are traveling toward or away from a light source, the wavelengths will be compressed or stretched and we can compute the difference using the doppler formula (8):

$$\begin{aligned}\frac{\Delta\lambda}{\lambda_0} &= \frac{v}{c} && (8) \\ \Delta\lambda &= \lambda_0 \times \frac{v}{c} \\ &= 1.063 \times 10^{-3}[\text{m}] \times \frac{1.555 \times 10^{6}[\text{m/s}]}{2.998 \times 10^{8}[\text{m/s}]} && (9) \\ &= 5.514 \times 10^{-6}\ [\text{m}] \\ &= 0.005514 \times 10^{-3}\ [\text{m}] && (10)\end{aligned}$$

Now that we have $\Delta\lambda$ (10) we know that when we are looking along our direction of motion, the wavelengths will be compressed or blue-shifted:

$$\begin{aligned}\lambda &= \lambda_0 - \Delta\lambda \\ &= 1.063 \times 10^{-3}\ [\text{m}] - 0.005514 \times 10^{-3}\ [\text{m}] \\ &= 1.057 \times 10^{-3}\ [\text{m}] && (11)\end{aligned}$$

3.c. **For the observer moving at 1555 km/s with respect to the cosmic background radiation, what is the peak wavelength of the cosmic background radiation measured from a point on the sky in the direction which is opposite that of the direction of motion?** In (part 3.b) we have already calculated $\Delta\lambda$ (10).

Since we are looking at a part of the sky *away* from our direction of motion, that part of the sky is traveling away from us and the wavelengths will be longer, or red-shifted.

$$\begin{aligned} \lambda &= \lambda_0 - \Delta\lambda \\ &= 1.063 \times 10^{-3}\ [\mathrm{m}] + 0.005514 \times 10^{-3}\ [\mathrm{m}] \\ &= 1.069 \times 10^{-3}\ [\mathrm{m}] \end{aligned} \tag{12}$$

3.d. **For the observer moving at 1555 km/s with respect to the cosmic background radiation, what is the peak wavelength of the cosmic background radiation, in a direction which is 90 degrees (at right angles) from the direction of motion?** If we look at a 90° angle to our direction of motion, then we will not be travelling either toward or away from that light. It will not be doppler shifted at all and so we will measure the same wavelength as in (part 3.a): 1.063×10^{-3} [m].