

HOMEWORK PROBLEMS:

1. (50 points)

An alien spaceship zooms past earth at a high speed which we measure to be 99.5% the speed of light. We monitor the ship and observe that it eventually reaches the Alpha Centauri star system, which is at distance of 4.37 light years from Earth.

- a.) How long does the trip take as measured by an observer on the earth?
- b.) How long does the trip take as measured by the aliens on the ship?
- c.) What is the distance from the earth to Alpha Centauri (in light years) as measured by the aliens on the ship?
- d.) What do the aliens measure as their velocity for the trip between Earth and Alpha Centauri?

[HINTs for part b: Study Fig 21b and Box 21-1 (10th edition) (or Fig 22b and Box 22-1 in 8th-9th edition)]

AND...According to us, how much time would elapse on our clocks during each second it takes for the alien's space-ship-clock to tick off 1 second?

AND...When dealing with time dilation, the rest frame should be the frame of rest with respect to the clock used in the measurement!]

2. (25 points)

G2 is an object near the Galactic Center (of our Milky Way Galaxy) that made a close approach to the central black hole in March 2014. It survived the close approach without being entirely disrupted, so we think it must be a star (or binary) with a surrounding gas cloud rather than simply a gas cloud.

- a.) Suppose G2 started out at a very large distance from the black hole with a velocity of zero. When it made its closest approach, it was at a distance of only $R=3000 R_{\text{Sch}}$ from the black hole. How fast will it be moving at its closest approach? Give the answer in both km/sec, and as a fraction of the speed of light. [Assume that all the mass is in the black hole. Ignore the mass from the rest of the galaxy.]
- b.) How much slower will time pass for any inhabitants of G2 (relative to us) at the time of closest approach, from the effects of special relativity?

3. (25 points)

The first-ever detection of gravitational waves announced in February 2016 is interpreted to originate from the merger of 2 stellar black holes. From the detailed fit to the shape of the gravitational wave, we think the original black holes had masses of $29 M_{\text{sun}}$ and $36 M_{\text{sun}}$, and the resulting (merged) black hole has a mass of $62 M_{\text{sun}}$.

a.) What percentage of the original mass is lost in the merger? What happened to the missing mass?

b.) To what density must the matter of a dead $36 M_{\text{sun}}$ star be compressed in order for the star to disappear inside its event horizon? How does this compare with the density at the center of a neutron star, about $3 \times 10^{18} \text{ kg m}^{-3}$?