HOMEWORK PROBLEMS:

1. (50 points) If 2 galaxies collide, almost none of the stars within them will collide because the distance between the stars is so large compared to the size of stars. Demonstrate that star-star collisions will be rare in the upcoming collision between the Milky Way Galaxy and the Andromeda Galaxy (M31). Assume that all the stars in both galaxies have the same size, and that the Milky Way and Andromeda are the same size.

NECESSARY BACKGROUND:

The sun will collide with another star of the same size (radius = 1 R_{sun}) if it comes within a distance of $2R_{sun}$ of that star. (Their edges will just touch if they come within a distance of 2 R_{sun}). The cross sectional area for a collision is then $\pi*distance^2 = \pi*(2xR_{sun})^2 = 4\pi*R_{sun}^2$.
(In other words, you can think of the stars in the "target galaxy" as targets with twice the diameter of the stars. If the center of a star from the "intruder galaxy" passes within the target area, then a collision occurs.)

The probability for 1 star passing through a galaxy to collide with any 1 particular star from that galaxy is given by the ratio of the cross sectional area for a collision with the projected area of the galaxy.
(This is so under the assumption that whether a star passing through the galaxy happens to encounter another star is a random event --in other words, that all paths through the galaxy are equally likely.)

a.) Calculate the projected (face-on) area of the large spiral galaxy M31, assuming that it has a radius of 50,000 light years. (Give area in square km.)

b.) Calculate the probability that 1 star (e.g., our Sun) passing through the "target galaxy" M31 collides with 1 PARTICULAR STAR in the "target galaxy".

c.) Calculate the probability that 1 star (e.g. our Sun) passing through the "target galaxy" M31 collides with ANY of the 100 billions stars in the M31.

d.) Calculate the probability that any of the 100 billions stars in the "intruder galaxy" (the Milky Way) collides with ANY of the 100 billions stars in the "target galaxy" M31.
[HINT: Each of the 100 billion stars in the "intruder galaxy" has the same probability as calculated in part c.]

2. (22 points)

a.) Calculate the escape velocity from the surface of the earth (in km/s, to 2 significant digits).

b.) Calculate the escape velocity (in km/s, to 2 significant digits) from the solar system, starting from a distance of 1.0 A.U. Assume that all the mass of the solar system is in the Sun. Assume that you do not have to also escape earth's gravity.

c.) Calculate the escape velocity (in km/s, to 2 significant digits) from the Milky Way Galaxy, starting from the position of the Sun. Assume that the total mass of the Milky Way is located inside a radius of 7.9 kiloparsecs, the orbital radius of the sun, and that this mass is 2.0×10^{11} M_{solar}. (This is an oversimplification since in reality there is mass in the Galaxy beyond the radius of the Sun!) Assume that you have already escaped from earth and the solar system.

3. (28 points) One the interesting things about black holes is that bigger ones have lower densities. Calculate BOTH the Schwarzschild radii and the densities (in kilograms per cubic meter) of black holes for the following masses:

[By density, I mean mass per unit volume inside the Schwarzschild radius. Even though all the mass is really at the central "singularity", it is interesting to know the average density inside the Schwarzschild radius, since that affects the behavior of matter just outside the Schwarzschild radius.]

[It is fine to show the full calculation for just one of the following, and to put the answers for all in a table.]

a.) Human Being (70 kg)
b.) Sun (2.0×10^{30} kg)
c.) Black hole in center of giant galaxy M87 (3×10^9 M_{solar})
d.) Cluster of Galaxies (2×10^{14} M_{solar})